

**Miriam Franchella**

## Some Reflections about Alain Badiou's Approach to Platonism in Mathematics

A reproach has been done many times to post-modernism: its picking up mathematical notions or results, mostly by misrepresenting their real content, in order to strike the readers and obtaining their assent only by impressing them (i. e. through the authority of mathematics itself). In this paper I intend to point out that although Alain Badiou's approach to philosophy starts with taking distance both from analytic philosophy and from French post-modernism, the categories that he uses for labelling logicism, formalism and intuitionism do not reflect the real content of the foundational schools. Hence, a re-thinking from him would be required about them; otherwise he would risk the same reproaches as post-modernists.

### *1. Introduction*

The beginning of Alain Badiou is foreign to both the analytic tradition and to the post-modernist tradition (in its post-structural expression), because his main question is: «how can a modern doctrine of the subject be reconciled with an ontology?»<sup>1</sup>. As it is stressed in the introduction to the English edition of some essays by him, appeared under the title *Infinite thought*, Badiou is convinced that the analytic tradition either forecloses ontology in favour of epistemology or reduces ontology to a property of theories<sup>2</sup>. The same introduction also recalls that post-structuralists seem to be not inclined to understand the above question: they cannot even distinguish a “subject” and an “object”, because there are no stable objects/subjects. Poststructuralists criticize the possibility of a substantial identity, and, by doing so, they leave open the problem of the differentiation between subjects and the problem of agency: if there is no self-identical subject, then what is the ground for autonomous rational action?

Still, Badiou's style recalls us, (it has so to say a taste of), postmodernism for his using some references to mathematics (in particular to foundational schools) inside his philosophical texts that do not respect the original conceptual content.

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<sup>1</sup> Badiou A. *L'être et l'événement*. – Paris: Éditions du Seuil, 1988, p. 10.

<sup>2</sup> In the footnote on p. 36 of Badiou A. *Infinite thoughts*. – London: Continuum, 2003, there is a reference to Willard Quine's essay “Ontological relativity” as an example of this tradition

We should recall here the polemic against postmodernists that has been raised by Alain Sokal, the physician that composed an article (“Transgressing the Boundaries: Toward a Transformative Hermeneutics of Quantum Gravity”) by collecting words into meaningless propositions and introducing here and there mathematical terms as a support for the theses that the paper was supposed to present. He submitted it to one of the most accredited journals of postmodernism *Social Text* and it was published. Sokal stressed<sup>3</sup> this fact as a proof of the fact that the style of thought of post-modernists was a simple production of chain of words, without care of their meaning, where the use of mathematical references is never clearly understood but is accepted (and, on the author’s side, is used) as a kind of authority argument.

As for himself, Badiou affirms<sup>4</sup>:

I claim the right to quote instances of mathematical reasoning, provided they are appropriate to the philosophical theses in the context of which they are being inscribed, and the knowledge required for understanding them has already been made available to the reader (p. 17).

He even put a glossary at the end of his books and invite those who do not understand to contact him. The addressees of these statements are not, however, Sokal’s friends, but the philosophers of the *little style*, those who believe that mathematics should not enter in philosophy. He reproaches them, by complaining that they understand a fragment by Anaximander, an elegy by Rilke, a seminar on the real by Lacan, but not the 2500-year-old proof that there are an infinity of prime numbers. So far, so good.

The problems arise from the fact that in Badiou we find some references to mathematics, in particular to set theory, that give him the support for affirming that “mathematics is ontology”. From this statement he derives the necessity of re-defining “mathematical Platonism”, by posing it in contraposition to a “mathematical Aristotelianism-Leibnizism”. Finally, he puts set-theory under the label “Platonism” and logicism, formalism, intuitionism under the label “Aristotelianism”. As we will see, *this distribution of foundational school is meaningless*.

## 2. Badiou’s references to set-theory

Let us consider what parts of mathematics he uses inside his exposition. *In primis* set theory, as it exemplifies his thought about ontology: he calls his re-

<sup>3</sup> See Sokal A., Bricmont J. Intellectual impostures. – London: Profile books, 1998.

<sup>4</sup> Badiou A. Theoretical writings. – London: Continuum, 2004.

reading of set-theory in ontological terms “meta-ontology”. He devotes large space to its treatment. We will recall here only a few main points, just to give a general idea of how it is developed.

The well-established conscience of the inconsistency of a class of all classes fits Badiou’s doctrine affirming that the “being of situations” is an inconsistent multiplicity, i. e. it cannot be thought of as a unity<sup>5</sup>. This fact forces us to describe the series of loose multiplicities only in negative terms, because we cannot speak of them as a unity. Within this framework, the metaphor of the void can help us, intended as something that we need but cannot describe. Badiou defines it as “*subtractive suture to being of a situation*”<sup>6</sup>: suture because it is the point of contact with being, and subtractive, because it does not possess the qualities of the situation neither can it be further described. The empty set at the basis of the set theoretical construction of sets seems to fit this use of the void. Furthermore, the axiom of separation, that can give properties (and hence describe sets) only provided some classes on which isolating elements, is interpreted as the statement that there must be a pre-existing multiplicity, a so-called “being in excess” (with respect to language) on which language can choose something<sup>7</sup>. Finally, set theory (at least in its version accepting Cohen’s axiom) can explain conceptual changes inside the universe. Cohen’s generic sets, that he uses to challenge the problem of continuum hypotheses, can explain such changes. A generic (sub-) set is only present at the level of inclusion and, unlike all the other subsets, cannot be known through its properties: whatever property is given, the generic set has at least one element that does not share that property. The relation that this subset entertains with the starting set is neither of pure exteriority nor of subsumption but that of indiscernibility<sup>8</sup>. In fact, none of the categories of the starting conceptualization can discern the nature of the new conceptualization: it is something like the paradigm change in Kuhn’s description of the evolution alongside the history of science.

On this basis, i. e. by passing thought his re-interpretation of set-theoretic axioms, Badiou can state “Mathematics is ontology”, because set-theory fits his viewpoint about metaphysics.

### 3. *Mathematical Platonism*

The belief that mathematics is ontology is also called by Badiou “the platonic gesture”. This belief has some interesting consequences. The most important

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<sup>5</sup> Badiou A. *L’être et l’événement*. – Paris: Éditions du Seuil, 1988, p. 36.

<sup>6</sup> *Ibid.*, p. 68.

<sup>7</sup> *Ibid.*, p. 58.

<sup>8</sup> *Ibid.*, p. 466.

for us is the necessity of destroying the present *cliché* of mathematical Platonism as defined in the Benacerraf-Putnam anthology<sup>9</sup>:

Platonists are people who consider mathematics as the discovery of truths about structures existing independently of the activity or mathematical thought (p. 62)

This cliché presupposes the distinction between a knowing subject and a known object. Still – Badiou notes<sup>10</sup> – Plato inherited from Parmenides the immanent identity of thought and being: Meno’s metaphor of reminiscence just stresses an identification of Ideas with thought, not their separation (p. 63). The problem of confronting a knowing subject with an external object comes from empiricists. As for Plato, he makes every effort even to find some trace of mathematical ideas in the “less educated, the more anonymous” thought: that of a slave. It should however be noticed, says Badiou – that Plato is not interested in the status of mathematical objects, but only in the movement of thought: mathematics is touched upon only insofar as it is useful to identify dialectics by differentiating this from mathematics.

What is this new definition of Platonism? Platonism admits that mathematics is thought,

...intransitive for sensitive and linguistic experience; it depends on a decision, it leaves place to the undecidable and assumes that whatever is consistent exists (p. 64).

Mathematics is “thought, intransitive to language and sense experience” means that mathematics is neither a formal nor an empirical discipline. This would be Plato viewpoint, as it would be testified by the fact that the realm of mathematics was for him a realm of ideas, whatever status one intends to attribute to them.

The characteristic of the undecidability is linked to the non-linguistic aspect of mathematics: nothing grants us that a thinkable entity always corresponds to a well-defined formula:

A platonist does not trust on the clarity of language in order to establish an existence (p. 64).

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<sup>9</sup> Badiou A. Platon et/ou Aristote-Leibniz. Théorie des ensembles et théorie des topos sous l’œil du philosophe in Panza M. // J.-M. Salanskis (ed.). L’objectivité mathématique. – Paris: Masson, 1995.

<sup>10</sup> Ibid.

Such characteristic is present already inside platonic dialogues: their aporetic style is used just to express the fact that thought is

neither a description nor a construction, but a rupture (with the opinion, with the experience), and hence a decision (p.64).

It is not descriptive, because it is

intransitive with respect to the opinion, hence to experience (p. 68)

Inside mathematical systems, we can imagine, a rupture means going outside the axioms, admitting that they cannot help us and establish ourselves which formulas are to be added. As a consequence, Badiou affirms that Zermelo isolation axiom is a very Platonist axiom, as it expresses the fact that we can claim the existence of a set in correspondence of a property only if the objects of the sets are already given. This characteristic is seen positively, as a continuous stimulus to invent through intuition. Furthermore, it shows us that mathematics – in particular, set theory, fits well in Badiou's general perspective that insisted on the subject a rupture-agent. Namely, Badiou defines the emergence of subject as the moment in which a human being encounters an event in his/her life that *disrupts* the situation in which he/she finds himself/herself and begins to act consequently with such an event (in “*fidelity*” with it). Nothing can force them to become subjects and they remain subjects till they remain faithful to the event. Hence, not all human beings are subjects and not throughout all their life. Doing mathematics, deciding about axioms is typical of subjects, because it is a kind of rupture. This is the reason why Badiou saw his perspective as touching mathematics, art, politics and love at the same time, because in each of these fields he stressed the relevance of “rupture”.

The last characteristic of the new-defined Platonism expresses the maximum admissible of freedom of thought: the only limit that we cannot avoid is the consistency of what is thought. As a consequence, one should accept only those axioms allowing the maximal extension of consistent thinkable: for instance, the axiom of choice should be accepted, because “the universe with the axiom of choice is larger and dense with meaningful links that an universe without it”. On the contrary, the continuum hypothesis and the constructability hypothesis are not accepted, because the related universe is poor.

Badiou finds out also similarities between some quotes from Plato and some characteristics of set-theory. The ‘*primitive name of being*’ in set theory is the empty set, in which all the hierarchy has its roots. Its existence is de-

cided, is posed exactly as, inside the *Parmenides*, it is shown as an apory the deduction of the existence of ‘One’: it is just fixed. Furthermore, set theory admits that differences between sets are “localized”, are well established: if two sets are different, this means that at least one of them has some elements that do not belong to the other.

#### 4. *Mathematical Aristotelism-Leibnizism*

The opposite of Platonism is what Badiou calls<sup>11</sup> “Aristotelism-Leibnizism”:

The core of the Aristotelian attitude towards mathematics is denying that it is a thought (pp. 67-69).

This means that mathematical knowledge is free from “the principle of being” (be it metaphysical or empirical), so it is meaningless to ask about its truth. One asks about its origin and verifies it. Its statements are analytic; hence they do not touch the “singularity always synthetic” of what exists. Mathematics is merely formal, hence it can only check according to rules: it is the contrary of freedom of thought. Hence, while a Platonist will choose the axioms that allow the maximum of existence, the Aristotelian will choose “logical prudences”. Actual infinite is avoided, otherwise “ties would be knot again with being”. Many mathematics are allowed: there is a tendence towards a pluralist perspectivism. While the Platonics are interested in axioms (that require decisions), the Aristotelians are interested in definitions.

As Badiou himself recalls in his *Theoretical Writings*<sup>12</sup>, the contraposition Platonism/Aristotelianism and his preference for Platonism let him for some time not appreciate the role of *mathematical* logic. Logic’s formal character, that could be considered the bridge between ancient logic and mathematical logic, was not judged by him a sufficient reason for such change

either this thesis comes up against the fact that mathematization has given a formidable impetus to logic, which contradicts the immutability supposedly imposed on it by its formal character; or it assumes that mathematics itself is purely formal, which in turn demands that we ask what distinguishes it from logic (p. 165).

Formal character meant for Badiou that logic had no links with ontology and was only something linguistic. On the contrary, inside his framework, logic, in order to be called “mathematical” should keep a link with ontology. He af-

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<sup>11</sup> Ibid.

<sup>12</sup> Badiou A. *Theoretical writings*. – London: Continuum, 2004.

firmed that the essence of mathematics was in no way formalization. Mathematics is a thought of “being qua being”:

Its formal transparency is a direct consequence of the absolutely univocal character of being. Mathematical writing is the transcription or inscription of this univocity (p. 173).

Hence, Badiou has denied the formal character of (mathematical) logic, because such character would keep it separated from ontology. Later, he fixed two conditions in order logic to be called mathematical:

a. Logic must emerge from within the movement of mathematics itself and not as the will to establish an extrinsic linguistic framework for mathematical activity. In giving birth to the ontological theory of sets, Cantor was not preoccupied with general and extrinsic aims, but with problems that were intrinsic to topology and classification of real numbers;

b. Logic must not be pegged to grammatical and linguistic analysis; its primary question must not be that of propositions, judgements or predicates. Logic must primarily provide a mathematical conception of the being of a universe of relations; or tell us what a possible situation of being is, when it is thought in its relational cohesion; or again, what being-there is, as the bound essence of the ineluctable localization of being (p. 173).

He also recalled that ontology is local, because the whole being cannot be thought of. Hence, there is place for some logic that describes the different ontologies, the possible universes, the *appearances*. The real “*mathematical*” logic, according to this perspective, would be category theory, specifically the theory of topoi. In this way, logic has a link with ontology but (with respect to Aristotle) also to mathematics. Badiou defines himself “ultra-Platonist” because he affirms that ontology is *nothing other* than mathematics and “citra-Platonist” because he does not diminish logic’s role. He stresses also his agreement with Leibniz’ viewpoint: logic is what is valid for every possible universe, by recalling his disagreement as for what concerns the universe as a whole: while Leibniz conceived it as governed “by any harmony or principle of reason”, Badiou sees it “as disseminated into an inconsistent, groundless multiple” (p. 172).

### 5. First criticisms

In this way, Badiou can recognise a meaning for the adjective “*mathematical*” when referred to a peculiar discipline (the theory of topoi) that he labelled as

“logic”. We can even see something more, by better analysing his position. He not only refuses to apply the adjective “*mathematical*” to logic if the above conditions are not satisfied, but he even denies the name of logic to formal logic

We should observe that, in this description, he often shifts the terms logic/theory of logic. He justifies this fact in his *Court traité d'ontologie transitoire*<sup>13</sup>, by considering it as an example of interplay locale/globale, by recalling implicitly this kind of dialectics as it was proposed in the ‘40s by Cavaillès: “theory of logic” is the global aspect, while the inner logic is the local aspect. In such case, each example of interplay local/global was considered a specification of the general dialectics present in human thought that relates opposites. Still, this is not enough to make acceptable the whole discourse by Badiou. Namely, the existence of different –loose – multiples opens spaces to an inner logic for each of them. What kind of logic? A set of characteristics? No, it does not seem. Here, Badiou speaks of the law of excluded middle, hence of what we usually know as logic. These laws may hold in a multitude and not in another; he refers to a specific ontology, still it is the usual “discursive” laws that has the original sin of having a linguistic nature (or a linguistic flavour). What seems to be “acceptable” inside his framework as “mathematical logic” is just that “theory of logic” that is the theory of topoi. But accepting it, as describing something ontological, seems to imply the acceptance of the “inner logics” of the ontologies insofar as they are described by the theory of topoi. It seems that the acceptance of the theory of topoi as “pure” mathematical logic (free from the original sin of being linguistic) implies the acceptance of different logics with their linguistic aspects. It cannot be replied that such logics are accepted but not as “mathematical”, because Badiou gives the label mathematical exactly to what he allows.

## 6. Labeling foundational schools

Badiou puts under the label “Aristotelian” the following trends: logicism, (algorithmic or constructivist) finitism and a not well-identifiable “pluralism of the rational possible”<sup>14</sup>. And this is the crucial point. How is it possible to find a label under which to collect together logicism, finitism and intuitionism (if not the general label *foundational schools*?)? Here we do not have to think of Platonism as the belief in the existence of mathematical entities independently of and outside men. Hence, there is no problem with the fact that logicism is

<sup>13</sup> Badiou A. *Court traité d'ontologie transitoire*. – Paris: Éditions du Seuil, 1998.

<sup>14</sup> Badiou A. Platon et/ou Aristote-Leibniz. Théorie des ensembles et théorie des topos sous l’œil du philosophe in Panza M. // J.-M. Salanskis (ed.). *L’objectivité mathématique*. – Paris: Masson, 1995, p. 69.



considered non-platonist, while its founder, Frege is commonly considered as a platonist in the usual sense. Still, Platonism is meant as the statement that mathematics is thought, where thought has nothing to do with language. Now, this is a fundamental basis for intuitionism. Brouwer has shown many times that mathematics is thought. Namely, we have mathematical attitude: we start<sup>15</sup> with ‘*mathematical observation*’ (*mathematische Betrachtung*) consisting of a ‘*temporal attitude*’ (i. e. perceiving sequences of instants) and a ‘*causal attitude*’ (i. e. identifying some parts of the sequences of appearances – *Erscheinungsfolgen* – and attributing them a common substratum) (p. 417). Then, we pass to mathematical abstraction, consisting of identifying the common substratum of all two-ities and then using it in order to build a mathematical system into which causal sequences can be projected (in order to be easily dominated) (p. 419). All of this aims to govern nature, hence it is judged negatively by Brouwer, because, from his perspective, happiness can be reached only in the Inner Self (pp. 1-2), hence anything that let man to go outside the inner Self is to be condemned. Still, if we keep only to two-ities, we can develop mathematics in a way which is morally acceptable. It has to remain inside the Self, i. e. it has to develop from intuition of time, by passing from some mental constructions to other mental constructions, without applicative aims.

Brouwer’s pupil, Arend Heyting, that did not want to keep linked intuitionism and Brouwer’s mysticism has tried to explain that intuitionistic mathematics needed no specific philosophical framework as it was based on our capability of isolating instants, which is *at the root of our thought*<sup>16</sup>. Hence, also in Heyting’s presentation of intuitionism, mathematics is seen as thought.

Badiou’s language-phobia is shared by Brouwer. The reason is always his mystical perspective, that sees any attempt to go outside man as a sin: language aims at communicating, hence at going outside. In particular, some mathematics that could be morally accepted, inside his perspective, had to be developed without using language, as an inner experience, where the *freedom of thought* could show itself<sup>17</sup>:

Mathematics is a free action independent of experience (p.97).

Mathematics did not need logic, as it did not need rules. We can recall here also Heyting’s statement<sup>18</sup>:

<sup>15</sup> Brouwer L. E. J. Collected Works. Vol. I. - Amsterdam: North-Holland, 1971.

<sup>16</sup> Heyting A. Intuitionistic Views on the Nature of Mathematics, Bollettino dell’UMI 9, p. 123.

<sup>17</sup> Brouwer L. E. J. Collected Works. Vol. I. – Amsterdam: North-Holland, 1971.

<sup>18</sup> Heyting A. Intuitionnisme, théorie de la démonstration. – Paris: Gauthier-Villars, 1935.

An exact enumeration of basic concepts and elementary conclusions admissible in intuitionistic mathematics is not enough as a basis for intuitionistic mathematics, because [...] it is absurd in itself to close the possibilities of thought inside a frame of principles of construction fixed in advance (p.14).

Logic was seen as something linguistic by intuitionists, something that needed a reference to the mental content of mathematics in order to be judged as true. On the purpose of logical principles, Brouwer asked himself<sup>19</sup>:

Can we have confidence that each part of the argument can be justified by recalling to the mind the corresponding mathematical construction? (p. 109).

The reason why Brouwer had to refuse the excluded middle as logical law was just that it did not reflect any mental content. Hence, all the constraints (no use of excluded middle and of reduction ad absurdum) that intuitionism puts are not due to the will of limiting thought, but only to the fact that they do not express thought. Intuitionism sees the principle of excluded middle as “*langagiere*”, because it has no mental content. The constructions required by intuitionism are mental constructions of which thought consists. They are not something linguistic. And even the isolation axiom of Zermelo is accepted by intuitionism, for the same reasons alleged by Badiou: Brouwer’s definition of “species” i. e. of intuitionistic sets was “a property that only mathematical entities can possess” (p. 302). Therefore, we could affirm that intuitionism cannot be enclosed under the label Aristotelianism but under the label “Platonism”. But we have to reckon with the other consequence that Badiou has linked to Platonism: the acceptance of what allows the maximum of existence, with no other constraints than consistency. We have to explain how to concile this with intuitionism. Well, our answer is that this consequence is not a direct and necessary consequence of Badiou’s definition of Platonism. Or, better, it requires a further reflection about thought. What can thought admit if not only those things that it can build by itself? If we do not share this statement, we are obliged to appeal to language in order to give a status to what is not a product of a mental construction. The same holds about infinity. Intuitionism refuses the actual infinite as it cannot be reached by thought, as it is only verbal: just the contrary of what is stated by Badiou.

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<sup>19</sup> Brouwer L. E. J. *Collected Works*. Vol. I. – Amsterdam: North-Holland, 1971.

One can object that language at the end gained some place inside intuitionism, and hence that even this can be defined '*langagiere*'. Still, we have to recall here that references to language were admitted only as a support for memory and to communicate results (inside the perspective of Arend Heyting, that did not share Brouwer's mystical solipsism), hence not as a machine to produce mathematical truths<sup>20</sup> ("Die formalen Regeln", p.42).

### 7. Further perplexities

There are other perplexities inside Badiou's categorizations.

By reading that the only "limit" that thought must admit is consistency, we would immediately recall Hilbert, at least "the first Hilbert", that had not yet specified the finitistic requirement. In his *Mathematische Probleme* he stated that non-contradictoriness is the only criterion for mathematical existence. Inside this framework, he proposed to formalize mathematics in order to have easier the homework to proof consistency: inside a formalized system, one has just to fix some signs representative for contradiction (for example:  $1 \neq 1$ ), performing deductions from axioms according to the rules in order to establish whether such string appears or not. This was just a trick, a shorter path to the end. At least, this was the original Hilbert's thought. He did not mean anything '*langagiere*', he did not intend to assert that mathematic is something formal, deprived of any content. He just wanted to use a (supposed) quicker method. The charge of "formalism" came from Brouwer, in his inaugural lecture of 1912, where he criticized non-contradictoriness as the only criterion for existence<sup>21</sup> :

The question where mathematical exactness does exist is answered differently by the two sides [i. e. by intuitionists and formalists]; the intuitionist says: in the human intellect, the formalist says: on paper (p. 125).

Thinking of a mathematical object meant for him building it step by step inside the mind: their consistency would be a by-product, a consequence of this building. According to Brouwer, the need for a proof of consistency derived from the linguistic nature of the mathematics that Hilbert (and Cantor, before him) was doing. If one proceeds by making mental steps and taking note of what one sees, then there is no further requirement. It is only if one makes

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<sup>20</sup> Heyting A. Die formalen Regeln der intuitionistischen Logik // Sitzungsbericht der preussischen Akademie von Wissenschaften, physikalische-mathematische Klasse, 1930, pp. 42-56, 57-71, 158-169

<sup>21</sup> Brouwer L. E. J. Collected Works vol. I. – Amsterdam: North-Holland, 1971.

suppositions beyond the human capability of mental constructing, for instance by accepting actual infinite, that problems arise. Hilbert's reply began the second phase of his research project: he revealed that he shared Brouwer's opinion that only a part of traditional mathematics has a content – and that it does not require a proof of consistency-, still he refused to do without the rest. He suggested to consider it an ideal in Kantian sense and to accept it only provided that a proof of its consistency would be given by using only finitistic tools, i. e. by performing mental constructions. For the “second Hilbert”, i. e. the Hilbert that tried to find a sort of compromise with Brouwer's requirements, the superposition of the requirement for tools to be finitistic came from the acknowledgement that going beyond the finite was something ‘*langagiere*’, had no real content, hence, for this aspect, he could be labeled as “Aristotelian” (within Badiou's framework) Still, the ‘*langagiere*’ aspect was just the infinitistic aspect that Badiou attached to Platonism! Even Hilbert's reference to the sign – recall his motto “In the beginning was the sign” is all but something linguistical. The signs – as he himself specified as a sort of reply to Brouwer's criticisms – are<sup>22</sup>

...certain extra-logical concrete objects which exist intuitively as immediate experiences before all thought. If logical inference is to be certain, then these objects must be capable of being completely surveyed in all their parts, and their presentation, their difference, their succession (like the objects themselves) must exist for us immediately, intuitively, as something that cannot be reduced to something else. Because I take this standpoint, the objects of the number theory are for me the sings themselves, whose shape can be generally and certainly recognized by me – independently of space and time, of the special conditions of the production of the sign, and of insignificant differences in the finishes product (pp. 1121-1122).

He also specified<sup>23</sup>:

The apriori is nothing more and nothing less than a fundamental mode of thought, which I also call the *finite mode of thought*: something is already given in advance in our faculty of representation: certain ex-

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<sup>22</sup> Hilbert D. Neubegründung der Mathematik // Abhandlungen aus dem mathematischen Seminar der Hamburgischen Universität, 1, 1922, pp. 157-177; Engl. transl. in W.B. Ewald (ed.). From Kant to Hilbert. Vol. II. – Oxford: Clarendon Press, 1996, pp. 1115-1134.

<sup>23</sup> Hilbert D. Die Grundlegung der elementaren Zahlentheorie // Mathematische Annalen, 104, 1931, pp. 485-494; Engl. transl. in W.B. Ewald (ed.). From Kant to Hilbert. Vol. II. – Oxford: Clarendon Press, 1996, pp.1149-1157.

tra-logical concrete objects that exist intuitively as an immediate experience before all thought [...] In this way I believe myself to have recognized and characterized the third source of knowledge that accompanies experience and logic ( p. 1150.)

Something analogous holds for logicism: grounding arithmetics on logic can apparently be interpreted as reducing it to language, but logic was for Frege... thought, just thought<sup>24</sup>:

Logic can also be called a normative science [...] the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter. [...] The sense of an assertoric sentence I call a 'thought'. [...] The predicate 'true' applies to thoughts. [...] A thought does not belong specially to the persons who think it, as does an idea to the person who has it: everyone who grasps it encounters it in the same way, as the same thought". (...) It is of the essence of a thought to be non-temporal and non-spatial [...] other people understand by thought an act of thinking (...) such act cannot be true. [...] The metaphors that underlie the expressions we use when we speak of grasping a thought, of conceiving, lying hold of, seizing, understanding, of *capere*, *percipere*, *comprehendere*, *intelligere*, put the matter in essentially the right perspective. What is grasped, taken hold of, is already there and all we do is take possession of it (pp. 228-237).

After attributing this task to logic, Frege tried to show that all of arithmetics can be reduced to logic. He began by stating some general axioms: among them the famous V axiom that was linked to our capability of building sets in correspondence of properties. Such capability, accepted without limitations produced Russell's antinomy, but a circumscribed form of it is at the basis of Zermelo's set theory that Badiou recognized as typical for Platonism. Frege's search for an ideography, a language suitable for his project, aimed simply to obtain a language deprived of daily connotations: something that could allow to perform a deduction, by helping intellect to remain faithful to *its* laws<sup>25</sup>:

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<sup>24</sup> Frege G. *Logic* // Hermes, Kaulbach, Kambartel (eds.). *Nachgelassene Schriften*. – Hamburg: Meiner, 1969, pp. 137-163; Engl. transl. in M. Beaney (ed.). *The Frege Reader*. – Oxford: Blackwell, 1997, pp. 227-250.

<sup>25</sup> Frege G. *Begriffsschrift*. – Halle: Nebert, 1879; Engl. transl. in M. Beaney (ed.). *The Frege Reader*. – Oxford: Blackwell, 1997, pp. 47-79.

If it is a task of philosophy to break the power of words over the human mind, by uncovering illusions that through the use of language often almost unavoidable arise concerning the relations of concepts, by freeing thoughts from the taint of ordinary linguistic means of expression, then my *Begriffsschrift*, further developed for these purposes, can become a useful tool for philosophers (pp. 50-51).

Furthermore, there is no particular constraint. Maybe Badiou had in mind Russell's type-theory, but also in this case the constraints are finalized to avoid contradictions, hence to satisfy a requirement that Badiou himself had put.

Even the apparently minimalist condition put by the Platonist Badiou, i. e. non –contradictoriness, requires to be checked. And this act will bring with it some further conditions – the hated constraints – on the methods through which it is performed.

### 8. Final remarks

What can we say about Badiou? He has of course a very interesting attitude, at least in principle: i. e. stopping and see what mathematicians do. Namely, he stated that philosophy is no longer sovereign<sup>26</sup>:

It is as if philosophy has finally heard that cry addressed to it for decades, a cry voiced by so many artists, scientists, activists and lovers whose activities it had deafly appropriated from on high, the cry “shut up and listen”!!! (p. 33).

In fact, the debate about Platonism after Benacerraf challenge had the defect to be almost among philosophers. There were, of course, some exceptions, that felt themselves the need to come back to mathematical practice: for instance, Penelope Maddy, with her neo-naturalist turn<sup>27</sup>. Still, the most part of that debate remained intramural, among philosophers. Badiou chooses to stop the “invasive” attitude of philosophy. For the sake of truth, we should stress that he acknowledges set-theory because it fits into his perspective: we do not know whether he would be so keen to actual mathematics, if set theory would not fit into his perspective.

It is also interesting his appreciation for category theory, in particular for the theory of topoi, as a general perspective on “possible logics”, that provided a kind of conciliation between the two traditions that he had pointed out:

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<sup>26</sup> Badiou A. *Infinite thoughts*. – London: Continuum, 2003.

<sup>27</sup> See for instance her *Naturalism in Mathematics*. – Oxford: Clarendon Press, 1997.

the Platonistic and the Aristotelian. Still, we can arise some doubts about his attaching these labels to the various foundational schools. It does not seem to be possible that they are placed inside the “Aristotelian box”, because they do not consider formalization as the real nature of mathematics, but only a tool (in the case of Brouwer, a not trustable tool). Set-theory, the axiomatic set theory that Badiou labels as Platonism grew at Hilbert court. Philosophy can use mathematical results to support its viewpoint, to exemplify its concepts. It is a right of philosophers categorizing world. It is an admirable act of philosophical modesty to refrain from saying to scientists what they can or may do. But a further effort is needed: to consider in all details the historical origin of the foundational schools, the real thoughts of their fathers, and not the stereotypes of them that literature can transmit. If a developed category-label does not square with the existent schools, maybe some of its aspect has not considered thoroughly. In any case, the historical schools are to be considered, otherwise, the whole theorization does not follow the prescription: “*Shut up philosophy*”. And Badiou’s papers would resemble to the post-modernist ones.