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A Decidability Theorem

The aim of this investigation is to show that supposed assumptions in the original proof of Gödel's First incompleteness theorem allow to infer a decidability of formulas that were asserted as undecidable in the theorem.

Gödel's First incompleteness theorem reads as follows[1]:

Incompleteness theorem. *For every ω–consistent recursive class $\varkappa$ of formulas there are recursive class sings r, such that neither $\upsilon\ Gen\ r$ nor $Neg(\upsilon\ Gen\ r)$ belongs to $Flg(\varkappa)$ (where υ is the free variable of r).*

Let us express the theorem by means of the logical symbolization:

$$\forall\varkappa\Big[\big(recursive(\varkappa)\ \&\ {}_{\omega}consist(\varkappa)\big)\ \supset\ \exists r\Big(recursive(r)\supset \overline{(\upsilon\ Gen\ r)\ \varepsilon\ Flg(\varkappa)}\ \ \&\ \ \overline{(Neg(\upsilon\ Gen\ r))\ \varepsilon\ Flg(\varkappa)}\Big)\Big].$$

Because r does not occur as free in $recursive(\varkappa)$ and ${}_{\omega}consist(\varkappa)$, given above formal expression of the theorem may be rewritten as (using exportation):

$$\forall\varkappa,\ \exists r\Big[\big(recursive(\varkappa)\ \&\ {}_{\omega}consist(\varkappa)\ \&\ recursive(r)\big)\supset \overline{(\upsilon\ Gen\ r)\ \varepsilon\ Flg(\varkappa)}\ \ \&\ \ \overline{(Neg(\upsilon\ Gen\ r))\ \varepsilon\ Flg(\varkappa)}\Big].$$

Three members of conjunction in the antecedent of implication in last are assumptions[2]:

[1] Gödel K. On formally undecidable propositions of *Principia Mathematica* and related systems I // van Heijenoort J. (ed.) Frege and Gödel: Two fundamental texts in mathematical logic. – Cambridge, Massachusetts, 1970, p. 98.

[2] For the first and the second it is obviously from the original proof of Gödel's First incompleteness theorem (see Ibid., pp. 98–100) and also from the reconstruction of it that was made recently by author (the reconstruction is published in this volume of *Analytica*, see pp. 37–77 (in russian)). Concerning the third assumption note that the formula $recursive(r)$ results from $recursive(q)$ indeed (where q is a relation sign) and the last results from implicative assumption of the original proof $Q(x,y)\ \rightleftarrows\ q(u_1,u_2)$ (see the reconstruction). But this is not significant for goals of the paper.

1) $recursive(\varkappa)$; 2) $_{\omega}consist(\varkappa)$; 3) $recursive(r)$.

These assumptions allow to derive a decidability of undecidable formulas. Before we propose the proof of this result, let us turn to six supplementary lemmas[3].

Lemma 1. *If a class $\varkappa$ of formulas is recursive, then a class $Flg(\varkappa)$ of formulas is recursive also.*

Proof. Let us prove the Lemma 1 by means of induction on length of inference.

1.	$recursive(\varkappa)$	assumption
2.	$x \,\varepsilon\, Flg(\varkappa) \equiv x \,\varepsilon\, \varkappa \vee Ax(x) \vee$ $\vee\, (y, z \,\varepsilon\, Flg(\varkappa) \,\&\, Fl(x, y, z))$	definition

Basis. Let length of inference is 1. Then exist two cases (by 2): $x \,\varepsilon\, \varkappa$ and $Ax(x)$. We have:

3.	$recursive(x \,\varepsilon\, \varkappa)$	$\varkappa \equiv \{x \mid x \,\varepsilon\, \varkappa\}$, ass. 1
4.	$recursive(Ax(x))$	Def. 42

Further we have:

5.	$recursive(Fl(x, y, z))$	Def. 43

[3] The proofs of *lemmas* and *the decidability theorem* are given with the help of the symbolization technique of metamathematical predicates that was suggested by Gödel in his system *P*. It is assumed that this system and definitions of following notions and relations are known by readers (see Ibid.):
1) $recursive(\varkappa) \equiv \forall x(x \,\varepsilon\, \varkappa \vee \overline{x \,\varepsilon\, \varkappa})$;
2) *a class sign* is a formula (a combination of signs) that has the form $a(b)$, where b is a sign of type 1 (i.e. a variable of the natural numbers) and a a sign of type 2 (i.e. a class of numbers); or has one of forms $\sim (a)$, $(a) \vee (b)$, $x\Pi(a)$, where x may be any variable;
3) $x \,\varepsilon\, Flg(\varkappa) \equiv x \,\varepsilon\, \varkappa \vee Ax(x) \vee (y, z \,\varepsilon\, Flg(\varkappa) \,\&\, Fl(x, y, z))$;
4) $x \; Gen \; y \equiv x\Pi(y)$;
5) $Sb(x^{\upsilon}_{y}) \equiv Subst \; a\binom{\upsilon}{b}$;
6) $_{\omega}consist(\varkappa) \equiv \overline{\exists a \left(\forall n[Sb(a^{\upsilon}_{Z(n)}) \,\varepsilon\, Flg(\varkappa)] \,\&\, [Neg(\upsilon \; Gen \; a)] \,\varepsilon\, Flg(\varkappa)\right)}$;
7) $\forall R(recursive(R) \equiv decid(R))$;
8) $recursive(r) \equiv recursive(R) \,\&\, R \leftrightarrows r$ (where the sign '$\leftrightarrows$' means a relation of one–to–one correspondence between an arbitrary relation (class) R and its isomorphic relation sign (class sign) r);
9) $Sb\big(r^{\upsilon}_{Z(n)}\big) \equiv r\big(Z(n)\big)$;
10) $decid(R) \equiv \exists r\big(R \to Bew(r) \,\&\, \overline{R} \to Bew(Neg(r))\big)$,
and also it is known that with the help of 5^{th} definition and some substitutions from the scheme of axiom III.1 of the system *P*, – i.e., $\upsilon\Pi(a) \supset Subst \; a\binom{\upsilon}{c}$, – the axiom $\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})$ turns out.

6.	$recursive(Fl(y,p,q))$	5 by $x/y; y/p; z/q$
7.	$recursive(Fl(z,u,w))$	5 by $x/z; y/u; z/w$
8.	$recursive(x \varepsilon \varkappa \vee Ax(x))$	Theorem II[4], 3, 4
9.	$recursive(y \varepsilon \varkappa \vee Ax(y))$	8 by x/y
10.	$recursive(z \varepsilon \varkappa \vee Ax(z))$	8 by x/z
11.	$recursive\big((y \varepsilon \varkappa \vee Ax(y)) \,\&\, (z \varepsilon \varkappa \vee Ax(z))\big)$	Theorem II, 9, 10

Let length of inference is 2. Then exist four cases (by 2):

1) $y \varepsilon \varkappa$ and $z \varepsilon \varkappa$ and $Fl(x,y,z)$; 3) $Ax(y)$ and $z \varepsilon \varkappa$ and $Fl(x,y,z)$;
2) $y \varepsilon \varkappa$ and $Ax(z)$ and $Fl(x,y,z)$; 4) $Ax(y)$ and $Ax(z)$ and $Fl(x,y,z)$.

All these cases we have in the following step:

12. $recursive\big[\big((y \varepsilon \varkappa \vee Ax(y)) \,\&\, (z \varepsilon \varkappa \vee Ax(z))\big) \,\&$ $\& \, Fl(x,y,z)\big]$ — Theorem II, 11, 5

Inductive step. Let length of inference is $n+1$. Last formula x of this inference may be 1) $x \varepsilon \varkappa$; 2) $Ax(x)$; 3) $Fl(x,y,z)$, where y and z are either $y \varepsilon \varkappa$ or $Ax(y)$ ($z \varepsilon \varkappa$ or $Ax(z)$); 4i) $Fl(x,y,z)$, where y is either $y \varepsilon \varkappa$ or $Ax(y)$; and z is such, that there are some preceding formulas u and w for that was stated $Fl(z,u,w)$; 4ii) $Fl(x,y,z)$, where y is such, that there are some preceding formulas p and q for that was stated $Fl(y,p,q)$; and z is either $z \varepsilon \varkappa$ or $Ax(z)$; 4iii) $Fl(x,y,z)$, where y and z such, that there are some preceding formulas p, q, u and w for that was stated $Fl(y,p,q)$ and $Fl(z,u,w)$. First three cases we have on lines 3, 4 and 12. Let us incorporate cases 4i, 4ii and 4iii with case on line 12. Then using inductive assumption about existence of formulas p, q, u and w we have:

13. $recursive\Big[\Big(y \varepsilon \varkappa \vee Ax(y) \vee$ $\vee \big(p,q \varepsilon Flg(\varkappa) \,\&\, Fl(y,p,q)\big)\Big) \,\&$ $\& \Big(z \varepsilon \varkappa \vee Ax(z) \vee$ $\vee \big(u,w \varepsilon Flg(\varkappa) \,\&\, Fl(z,u,w)\big)\Big) \,\&$ $\& \, Fl(x,y,z)\Big]$ — Theorem II, 6, 7, 9, 10

14. $y \varepsilon Flg(\varkappa) \equiv y \varepsilon \varkappa \vee Ax(y) \vee$ $\vee (p,q \varepsilon Flg(\varkappa) \,\&\, Fl(y,p,q))$ — 2 by x/y

[4] Ibid., p. 93.

15.	$z \,\varepsilon\, Flg(\varkappa) \equiv\ z \,\varepsilon\, \varkappa\ \vee\ Ax(z) \vee$	2 by x/z
	$\vee\ (u, w \,\varepsilon\, Flg(\varkappa)\ \&\ Fl(z, u, w))$	
16.	$recursive\big(y \,\varepsilon\, Flg(\varkappa)\ \&\ z \,\varepsilon\, Flg(\varkappa)\ \&$	2–change rule, 13, 14, 15
	$\&\ Fl(x, y, z)\big)$	
17.	$recursive\big(x \,\varepsilon\, \varkappa\ \vee\ Ax(x) \vee$	Theorem II, 3, 4, 16
	$\big(y \,\varepsilon\, Flg(\varkappa)\ \&\ z \,\varepsilon\, Flg(\varkappa)\ \&\ Fl(x, y, z)\big)\big)$	
18.	$recursive(x \,\varepsilon\, Flg(\varkappa))$	change rule, 17, 2
19.	$recursive(Flg(\varkappa))$	$Flg(\varkappa) \equiv$
		$\{x \mid\ x \,\varepsilon\, Flg(\varkappa)\}$, 18
20.	$recursive(\varkappa) \supset recursive(Flg(\varkappa))$	ass. elim., 1

□

Lemma 2. *If a class $Flg(\varkappa)$ of formulas is recursive, then for the given class sign r can be defined whether it belongs to the class $Flg(\varkappa)$ or not.*

Proof.

1.	$recursive(\varkappa) \equiv \forall r(r \,\varepsilon\, \varkappa\ \vee\ \overline{r \,\varepsilon\, \varkappa})$	definition
2.	$recursive(Flg(\varkappa)) \equiv \forall r(r \,\varepsilon\, Flg(\varkappa)\ \vee\ \overline{r \,\varepsilon\, Flg(\varkappa)})$	1 by $\varkappa/Flg(\varkappa)$
3.	$\forall r(A(r)) \equiv\ A(r)$	logic rule
4.	$\forall r(r \,\varepsilon\, Flg(\varkappa)\ \vee\ \overline{r \,\varepsilon\, Flg(\varkappa)}) \equiv$	2 by $A(r)/r \,\varepsilon\, Flg(\varkappa) \vee$
	$\equiv\ r \,\varepsilon\, Flg(\varkappa)\ \vee\ \overline{r \,\varepsilon\, Flg(\varkappa)}$	$\vee\, \overline{r \,\varepsilon\, Flg(\varkappa)}$
5.	$recursive(Flg(\varkappa)) \equiv\ r \,\varepsilon\, Flg(\varkappa)\ \vee\ \overline{r \,\varepsilon\, Flg(\varkappa)}$	transitivity of $\equiv$, 2, 4
6.	$recursive(Flg(\varkappa)) \supset\ r \,\varepsilon\, Flg(\varkappa)\ \vee\ \overline{r \,\varepsilon\, Flg(\varkappa)}$	$\equiv$–elim., 5

□

Lemma 3. (*i*) *If the given class sign r belongs to a class $Flg(\varkappa)$, then υ Gen r belongs to the same class;* (*ii*) *If the class sign r does not belong to the class $Flg(\varkappa)$, then υ Gen r also does not belong to the same class.*

Proof.
(*i*)

1.	$r \,\varepsilon\, Flg(\varkappa)$	assumption

2.	$\upsilon\, Gen\, r \equiv\ r$	logic rule
3.	$\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	change rule, 1, 2
4.	$r\, \varepsilon\, Flg(\varkappa)\ \supset\ \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	⊃–enter., ass. elim., 1

(*ii*)

1.	$\overline{r\, \varepsilon\, Flg(\varkappa)}$	assumption
2.	$\upsilon\, Gen\, r \equiv\ r$	logic rule
3.	$\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}$	change rule, 1, 2
4.	$\overline{r\, \varepsilon\, Flg(\varkappa)}\ \supset\ \overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}$	⊃–enter., ass. elim., 1

□

Lemma 4. *Suppose $\upsilon\ Gen\ r$ belongs to a class $Flg(\varkappa)$. Let $Sb(r^{\upsilon}_{Z(n)})$ be a formula that results from a class sign r by a substitution for its free variable υ by a numeral of the number n; then does not exist a number n, such that $Sb(r^{\upsilon}_{Z(n)})$ does not belong to the class $Flg(\varkappa)$.*

Proof.

1.	$\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	assumption
2.	$\upsilon\, Gen\, r\ \equiv\ \upsilon\Pi(r)$	definition
3.	$\upsilon\Pi(r)\, \varepsilon\, Flg(\varkappa)$	change rule, 2, 1
4.	$\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})$	axiom
5.	$Ax\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)$	Def. 42, 38
6.	$Ax\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\ \supset$ $Ax\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\ \vee\ \big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\, \varepsilon\, \varkappa \vee$ $\big(y, z\, \varepsilon\, Flg(\varkappa)\ \&\ Fl\big(\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big), y, z\big)\big)$	logic rule
7.	$Ax\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\ \vee\ \big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\, \varepsilon\, \varkappa \vee$ $\big(y, z\, \varepsilon\, Flg(\varkappa)\ \&\ Fl\big(\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big), y, z\big)\big)$	*m.p.*, 6, 5
8.	$\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\, \varepsilon\, Flg(\varkappa) \equiv$	definition

$Ax\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\ \vee\ \big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ \varkappa \vee$
$\big(y, z\ \varepsilon\ Flg(\varkappa)\ \&\ Fl\big(\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big), y, z\big)\big)$

9. $Ax\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\ \vee\ \big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ \varkappa \vee$ ≡–elim., 8
$\big(y, z\ \varepsilon\ Flg(\varkappa)\ \&\ Fl\big(\big(\upsilon\Pi(r) \supset\ Sb(r^{\upsilon}_{Z(n)})\big), y, z\big)\big)\ \supset$
$\big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)$

10. $\big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)$ *m.p.*, 9, 7

11. $Fl\ \big(Sb(r^{\upsilon}_{Z(n)}),\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big),\ \upsilon\Pi(r)\big)$ Def. 43

12. $\upsilon\Pi(r)\ \varepsilon\ Flg(\varkappa)\ \&\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)\ \&$ &–enter., 3,
$Fl\ \big(Sb(r^{\upsilon}_{Z(n)}),\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big),\ \upsilon\Pi(r)\big)$ 10, 11

13. $\upsilon\Pi(r)\ \varepsilon\ Flg(\varkappa)\ \&\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)\ \&$ logic rule
$Fl\ \big(Sb(r^{\upsilon}_{Z(n)}),\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big),\ \upsilon\Pi(r)\big) \supset$
$\Big\{\upsilon\Pi(r)\ \varepsilon\ Flg(\varkappa)\ \&\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)\ \&$
$Fl\ \big(Sb(r^{\upsilon}_{Z(n)}),\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big),\ \upsilon\Pi(r)\big)\Big\}\vee$
$Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ \varkappa\ \vee\ Ax\big(Sb(r^{\upsilon}_{Z(n)})\big)$

14. $\Big\{\upsilon\Pi(r)\ \varepsilon\ Flg(\varkappa)\ \&\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)\ \&$ *m.p.*, 13, 12
$Fl\ \big(Sb(r^{\upsilon}_{Z(n)}),\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big),\ \upsilon\Pi(r)\big)\Big\}\vee$
$Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ \varkappa\ \vee\ Ax\big(Sb(r^{\upsilon}_{Z(n)})\big)$

15. $Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa) \equiv$ definition
$\Big\{\upsilon\Pi(r)\ \varepsilon\ Flg(\varkappa)\ \&\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)\ \&$
$Fl\ \big(Sb(r^{\upsilon}_{Z(n)}),\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big),\ \upsilon\Pi(r)\big)\Big\}\vee$
$Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ \varkappa\ \vee\ Ax\big(Sb(r^{\upsilon}_{Z(n)})\big)$

16. $\Big\{\upsilon\Pi(r)\ \varepsilon\ Flg(\varkappa)\ \&\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big)\ \varepsilon\ Flg(\varkappa)\ \&$ ≡–elim., 15
$Fl\ \big(Sb(r^{\upsilon}_{Z(n)}),\ \big(\upsilon\Pi(r) \supset Sb(r^{\upsilon}_{Z(n)})\big),\ \upsilon\Pi(r)\big)\Big\}\vee$
$Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ \varkappa\ \vee\ Ax\big(Sb(r^{\upsilon}_{Z(n)})\big)\ \supset$
$Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)$

17.	$Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)$	*m.p.*, 16, 14
18.	$\forall n\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big]$	$\forall$–enter., 17
19.	$\forall n\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big] \equiv \overline{\exists n\overline{\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big]}}$	logic rule
20.	$\forall n\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big] \supset \overline{\exists n\overline{\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big]}}$	$\equiv$–elim., 19
21.	$\overline{\exists n\overline{\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big]}}$	*m.p.*, 20, 18
22.	$\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa) \supset \overline{\exists n\overline{\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big]}}$	$\supset$–enter., ass. elim., 1

□

Lemma 5. *Suppose a class $\varkappa$ of formulas is ω–consistent. Let $Sb(r^{\upsilon}_{Z(n)})$ be a formula that results from a class sign r by a substitution for its free variable υ by a numeral of the number n; then either $Neg(\upsilon\ Gen\ r)$ does not belong to a class $Flg(\varkappa)$ or exists a number n, such that $Sb(r^{\upsilon}_{Z(n)})$ does not belong to the class $Flg(\varkappa)$.*

Proof.

1.	${}_{\omega}consist(\varkappa)$	assumption
2.	${}_{\omega}consist(\varkappa) \equiv \overline{\exists r\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)}$	definition
3.	${}_{\omega}consist(\varkappa) \supset \overline{\exists r\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)}$	$\equiv$–elim., 2
4.	$\overline{\exists r\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)}$	*m.p.*, 3, 1
5.	$\overline{\exists r\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)} \equiv \forall r\overline{\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)}$	logic rule
6.	$\overline{\exists r\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)} \supset \forall r\overline{\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)}$	$\equiv$–elim., 5
7.	$\forall r\overline{\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)}$	*m.p.*, 6, 4

8.	$\overline{\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)}$	$\forall$–elim., 7
9.	$\overline{\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)} \equiv \overline{\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]} \vee \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	A. de Morgan rule
10.	$\overline{\Big(\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\Big)} \supset \overline{\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]} \vee \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	$\equiv$–elim., 9
11.	$\overline{\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]} \vee \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	*m.p.*, 10, 8
12.	$\overline{\forall n[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]} \equiv \exists n\overline{[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]}$	logic rule
13.	$\exists n\overline{[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]} \vee \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	change rule, 12, 11
14.	${}_{\omega}consist(\varkappa) \supset \exists n\overline{[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]} \vee \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	$\supset$–enter., ass. elim., 1

□

Lemma 6. *Suppose a class sign r is recursive; then either $\upsilon\ Gen\ r$ or $Neg(\upsilon\ Gen\ r)$ belongs to a class $Flg(\varkappa)$.*

Proof.

1.	$recursive(r)$	assumption
2.	$recursive(r) \equiv recursive(R)\ \&\ R \leftrightarrows r$	definition
3.	$recursive(r) \supset recursive(R)\ \&\ R \leftrightarrows r$	$\equiv$–elim., 2
4.	$recursive(R)\ \&\ R \leftrightarrows r$	*m.p.*, 3, 1
5.	$recursive(R)\ \&\ R \leftrightarrows r \supset recursive(R)$	logic rule
6.	$recursive(R)\ \&\ R \leftrightarrows r \supset R \leftrightarrows r$	logic rule
7.	$recursive(R)$	*m.p.*, 5, 4
8.	$R \leftrightarrows r$	*m.p.*, 6, 4
9.	$\forall R(recursive(R) \equiv decid(R))$	definition[5]
10.	$recursive(R) \equiv decid(R)$	$\forall$–elim., 9
11.	$recursive(R) \supset decid(R)$	$\equiv$–elim., 10

[5] The corollary of Gödel's Theorem V (see Ibid., p. 100.)

12.	$decid(R)$	*m.p.*, 11, 7
13.	$decid(R) \equiv\ \exists r\big(R \to Bew(r)\ \&\ \overline{R} \to Bew(Neg(r))\big)$	definition
14.	$decid(R) \supset\ \exists r\big(R \to Bew(r)\ \&\ \overline{R} \to Bew(Neg(r))\big)$	≡–elim., 13
15.	$\exists r\big(R \to Bew(r)\ \&\ \overline{R} \to Bew(Neg(r))\big)$	*m.p.*, 14, 12
16.	$R \to Bew(r)\ \&\ \overline{R} \to Bew(Neg(r))$	∃–elim., 15, 8
17.	$Neg(R) \equiv \overline{R}$	definition
18.	$R \to Bew(r)\ \&\ Neg(R) \to Bew(Neg(r))$	change rule, 17, 16
19.	$Neg(Bew(r)) \to Neg(R)\ \&$ $Neg(Bew(Neg(r))) \to Neg(Neg(R))$	twice contra–position, 18
20.	$R \to Bew(r)\ \&\ Neg(R) \to Bew(Neg(r))\ \&$ $Neg(Bew(r)) \to Neg(R)\ \&$ $Neg(Bew(Neg(r))) \to Neg(Neg(R))$	&–enter., 18, 19
21.	$Neg(Bew(Neg(r))) \to Neg(Neg(R))\ \&\ R \to Bew(r)\ \&$ $Neg(Bew(r)) \to Neg(R)\ \&\ Neg(R) \to Bew(Neg(r))$	commutat. of &, 20
22.	$\big(Neg(Bew(Neg(r))) \to Neg(Neg(R))\ \&\ R \to Bew(r)\big)\ \&$ $\big(Neg(Bew(r)) \to Neg(R)\ \&\ Neg(R) \to Bew(Neg(r))\big)$	associativity of &, 21
23.	$Neg(Neg(R)) \equiv R$	logic rule
24.	$\big(Neg(Bew(Neg(r))) \to R\ \&\ R \to Bew(r)\big)\ \&$ $\big(Neg(Bew(r)) \to Neg(R)\ \&\ Neg(R) \to Bew(Neg(r))\big)$	change rule, 23, 22
25.	$\big(Neg(Bew(Neg(r))) \to Bew(r)\big)\ \&$ $\big(Neg(Bew(r)) \to Bew(Neg(r))\big)$	twice transiti–vity of →, 24
26.	$Neg(A) \to B \equiv A \vee B$	logic rule
27.	$Neg(Bew(Neg(r))) \to Bew(r) \equiv Bew(Neg(r)) \vee Bew(r)$	26 by $A/Bew(Neg(r))$, $B/Bew(r)$
28.	$Neg(Bew(r)) \to Bew(Neg(r)) \equiv Bew(r) \vee Bew(Neg(r))$	26 by $B/Bew(Neg(r))$, $A/Bew(r)$
29.	$\big(Bew(Neg(r)) \vee Bew(r)\big)\ \&\ \big(Bew(r) \vee Bew(Neg(r))\big)$	twice change rule, 27–28, 25

30.	$\big(Bew(Neg(r)) \vee Bew(r)\big) \,\&\, \big(Bew(r) \vee Bew(Neg(r))\big) \supset Bew(Neg(r)) \vee Bew(r)$	logic rule
31.	$Bew(Neg(r)) \vee Bew(r)$	*m.p.*, 30, 29
32.	$\upsilon\, Gen\, r \equiv\ r$	logic rule
33.	$Bew(Neg(\upsilon\, Gen\, r)) \vee Bew(\upsilon\, Gen\, r)$	change rule, 32, 31
34.	$\forall x[Bew(x) \rightarrow Bew_{\varkappa}(x)]$	statement (8)[6]
35.	$Bew(Neg(\upsilon\, Gen\, r)) \supset Bew_{\varkappa}(Neg(\upsilon\, Gen\, r))$	∀–elim., 34
36.	$Bew(\upsilon\, Gen\, r) \supset Bew_{\varkappa}(\upsilon\, Gen\, r)$	∀–elim., 34
37.	$\forall x[Bew_{\varkappa}(x) \equiv x\, \varepsilon\, Flg(\varkappa)]$	statement (7)[7]
38.	$Bew_{\varkappa}(Neg(\upsilon\, Gen\, r)) \equiv Neg(\upsilon\, Gen\, r)\, \varepsilon\, Flg(\varkappa)$	∀–elim., 37
39.	$Bew_{\varkappa}(Neg(\upsilon\, Gen\, r)) \supset Neg(\upsilon\, Gen\, r)\, \varepsilon\, Flg(\varkappa)$	≡–elim., 38
40.	$Bew_{\varkappa}(\upsilon\, Gen\, r) \equiv \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	∀–elim., 37
41.	$Bew_{\varkappa}(\upsilon\, Gen\, r) \supset \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	≡–elim., 40
42.	$Bew(Neg(\upsilon\, Gen\, r)) \supset Neg(\upsilon\, Gen\, r)\, \varepsilon\, Flg(\varkappa)$	transitivity of ⊃, 35, 39
43.	$Bew(\upsilon\, Gen\, r) \supset \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	transitivity of ⊃, 36, 41
44.	$Bew(Neg(\upsilon\, Gen\, r)) \vee Bew(\upsilon\, Gen\, r) \,\&\, Bew(Neg(\upsilon\, Gen\, r)) \supset Neg(\upsilon\, Gen\, r)\, \varepsilon\, Flg(\varkappa) \,\&\, Bew(\upsilon\, Gen\, r) \supset \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	&–enter., 33, 42–43
45.	$(A \vee B \,\&\, A \supset C \,\&\, B \supset D) \supset C \vee D$	logic rule
46.	$\Big(Bew(Neg(\upsilon\, Gen\, r)) \vee Bew(\upsilon\, Gen\, r) \,\&\, Bew(Neg(\upsilon\, Gen\, r)) \supset Neg(\upsilon\, Gen\, r)\, \varepsilon\, Flg(\varkappa) \,\&\, Bew(\upsilon\, Gen\, r) \supset \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\Big) \supset Neg(\upsilon\, Gen\, r)\, \varepsilon\, Flg(\varkappa) \vee \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	45 by $A/Bew(Neg(\upsilon\, Gen\, r))$, $B/Bew(\upsilon\, Gen\, r)$, $C/Neg(\upsilon\, Gen\, r)\, \varepsilon\, \varepsilon\, Flg(\varkappa)$, $D/\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$
47.	$Neg(\upsilon\, Gen\, r)\, \varepsilon\, Flg(\varkappa) \vee \upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)$	*m.p.*, 46, 44

[6] Ibid., p. 99.
[7] Ibid.

48.	$recursive(r) \supset$	$\supset$–enter.,
	$Neg(\upsilon\, Gen\, r)\ \varepsilon\ Flg(\varkappa)\ \vee\ \upsilon\, Gen\, r\ \varepsilon\ Flg(\varkappa)$	ass. elim., 1

$\square$

We now had been approaching to the main goal of present paper. The main result about the decidability of undecidable propositions consists in following:

Decidability theorem. *For every ω–consistent recursive class $\varkappa$ of formulas, for all recursive class sings r strictly either $\upsilon\ Gen\ r$ or $Neg(\upsilon\ Gen\ r)$ belongs to $Flg(\varkappa)$ (where υ is the free variable of r).*

Let us express the theorem symbolically:

$$\forall\varkappa, \forall r\Big[\Big(recursive(\varkappa)\ \&\ {}_{\omega}consist(\varkappa)\ \&\ recursive(r)\Big) \supset \Big(\big(\upsilon\, Gen\, r\ \varepsilon\ Flg(\varkappa)\ \vee\ [Neg(\upsilon\, Gen\, r)]\ \varepsilon\ Flg(\varkappa)\big)\ \&\ \big(\overline{\upsilon\, Gen\, r\ \varepsilon\ Flg(\varkappa)\ \&\ [Neg(\upsilon\, Gen\, r)]\ \varepsilon\ Flg(\varkappa)}\big)\ \&\ \big(\overline{\overline{\upsilon\, Gen\, r\ \varepsilon\ Flg(\varkappa)}\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\ \varepsilon\ Flg(\varkappa)}}\big)\Big)\Big].$$

Proof.

1.	$recursive(\varkappa)$	assumption 1
2.	${}_{\omega}consist(\varkappa)$	assumption 2
3.	$recursive(r)$	assumption 3
4.	$recursive(\varkappa) \supset recursive(Flg(\varkappa))$	Lemma 1
5.	$recursive(Flg(\varkappa))$	*m.p.*, 4, 1
6.	$recursive(Flg(\varkappa)) \supset\ r\ \varepsilon\ Flg(\varkappa)\ \vee\ \overline{r\ \varepsilon\ Flg(\varkappa)}$	Lemma 2
7.	$r\ \varepsilon\ Flg(\varkappa)\ \vee\ \overline{r\ \varepsilon\ Flg(\varkappa)}$	*m.p.*, 6, 5
8.	$r\ \varepsilon\ Flg(\varkappa)$	assumption 4
9.	$r\ \varepsilon\ Flg(\varkappa)\ \supset\ \upsilon\, Gen\, r\ \varepsilon\ Flg(\varkappa)$	Lemma 3(*i*)
10.	$\upsilon\, Gen\, r\ \varepsilon\ Flg(\varkappa)$	*m.p.*, 9, 8
11.	$\upsilon\, Gen\, r\ \varepsilon\ Flg(\varkappa) \supset \overline{\exists n\overline{\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big]}}$	Lemma 4
12.	$\overline{\exists n\overline{\big[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)\big]}}$	*m.p.*, 11, 10
13.	${}_{\omega}consist(\varkappa) \supset$	Lemma 5

	$\exists n\overline{[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]}\ \vee\ \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	
14.	$\exists n\overline{[Sb(r^{\upsilon}_{Z(n)})\ \varepsilon\ Flg(\varkappa)]}\ \vee\ \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	*m.p.*, 13, 2
15.	$\overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	resolution, 14, 12
16.	$\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)\ \&\ \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	&–enter., 10, 15
17.	$r\ \varepsilon\ Flg(\varkappa) \supset$ $\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)\ \&\ \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$	⊃–enter., ass. 4 elim., 8
18.	$\overline{r\ \varepsilon\ Flg(\varkappa)}$	assumption 5
19.	$\overline{r\ \varepsilon\ Flg(\varkappa)}\ \supset\ \overline{\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)}$	Lemma 3(*ii*)
20.	$\overline{\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)}$	*m.p.*, 19, 18
21.	$recursive(r) \supset$ $Neg(\upsilon\ Gen\ r)\ \varepsilon\ Flg(\varkappa)\ \vee\ \upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)$	Lemma 6
22.	$Neg(\upsilon\ Gen\ r)\ \varepsilon\ Flg(\varkappa)\ \vee\ \upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)$	*m.p.*, 21, 3
23.	$Neg(\upsilon\ Gen\ r)\ \varepsilon\ Flg(\varkappa)$	resolution, 22, 20
24.	$\overline{\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)}\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)$	&–enter., 20, 23
25.	$\overline{r\ \varepsilon\ Flg(\varkappa)} \supset$ $\overline{\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)}\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)$	⊃–enter., ass. 5 elim., 18
26.	$\big(r\ \varepsilon\ Flg(\varkappa)\ \vee\ \overline{r\ \varepsilon\ Flg(\varkappa)}\big)\ \&$ $\big(r\ \varepsilon\ Flg(\varkappa) \supset$ $\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)\ \&\ \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}\big)\ \&$ $\big(\overline{r\ \varepsilon\ Flg(\varkappa)} \supset$ $\overline{\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)}\ \&\ [Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)\big)$	&–enter., 7, 17, 25
27.	$\big((a\ \vee\ b)\ \&\ (a \supset c)\ \&\ (b \supset d)\big) \supset c\ \vee\ d$	logic rule
28.	$\Big(\big(r\ \varepsilon\ Flg(\varkappa)\ \vee\ \overline{r\ \varepsilon\ Flg(\varkappa)}\big)\ \&$ $\big(r\ \varepsilon\ Flg(\varkappa) \supset$ $\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)\ \&\ \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}\big)\ \&$ $\big(\overline{r\ \varepsilon\ Flg(\varkappa)} \supset$	27 by $a/r\ \varepsilon\ Flg(\varkappa)$; $b/\overline{r\ \varepsilon\ Flg(\varkappa)}$; $c/\upsilon\ Gen\ r\ \varepsilon\ Flg(\varkappa)\ \&$ $\&\ \overline{[Neg(\upsilon\ Gen\ r)]\ \varepsilon\ Flg(\varkappa)}$

$\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)\Big) \supset$

$\big(\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}\big) \vee$

$\vee \big(\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)$ — $d/\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)$

29. $\big(\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}\big) \vee$

$\vee \big(\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)$ — *m.p.*, 28, 26

30. $\big(\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \vee\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)\ \&$

$\big(\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}\big)\ \&$

$\big(\overline{\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}}\big)$ — distr. of $\vee$, distr. of &, A. de Morgan laws, &–enter., 29

31. $recursive(r) \supset$

$\Big(\big(\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \vee\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)\ \&$

$\big(\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}\big)\ \&$

$\big(\overline{\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}}\big)\Big)$ — $\supset$–enter., ass. 3 elim., 3, 30

32. ${}_{\omega}consist(\varkappa) \supset \Big(recursive(r) \supset$

$\Big(\big(\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \vee\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)\ \&$

$\big(\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}\big)\ \&$

$\big(\overline{\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}}\big)\Big)\Big)$ — $\supset$–enter., ass. 2 elim., 2, 31

33. $recursive(\varkappa) \supset \Big({}_{\omega}consist(\varkappa) \supset \Big(recursive(r) \supset$

$\Big(\big(\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \vee\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)\ \&$

$\big(\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}\big)\ \&$

$\big(\overline{\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}}\big)\Big)\Big)\Big)$ — $\supset$–enter., ass. 1 elim., 1, 32

34. $\Big(recursive(\varkappa)\ \&\ {}_{\omega}consist(\varkappa)\ \&\ recursive(r)\Big) \supset$

$\Big(\big(\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \vee\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)\big)\ \&$

$\big(\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)\ \&\ [Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}\big)\ \&$

$\big(\overline{\overline{\upsilon\, Gen\, r\, \varepsilon\, Flg(\varkappa)}\ \&\ \overline{[Neg(\upsilon\, Gen\, r)]\, \varepsilon\, Flg(\varkappa)}}\big)\Big)$ — importation, 33

35. $$\forall\varkappa, \forall r \Big[\Big(recursive(\varkappa) \ \& \ {}_{\omega}consist(\varkappa) \ \& \ recursive(r) \Big) \supset \Big(\big(\upsilon\, Gen\, r \ \varepsilon\ Flg(\varkappa) \ \vee \ [Neg(\upsilon\, Gen\, r)]\ \varepsilon\ Flg(\varkappa) \big) \ \& \ \big(\overline{\upsilon\, Gen\, r \ \varepsilon\ Flg(\varkappa) \ \& \ [Neg(\upsilon\, Gen\, r)]\ \varepsilon\ Flg(\varkappa)} \big) \ \& \ \big(\overline{\overline{\overline{\upsilon\, Gen\, r \ \varepsilon\ Flg(\varkappa)} \ \& \ \overline{[Neg(\upsilon\, Gen\, r)]\ \varepsilon\ Flg(\varkappa)}}} \big) \Big) \Big]$$ 2–∀–enter., 34

□

Q. E. D.